The Integration Problem in Physics

- Introducing integration in Physics, as a tool to solve inhomogeneous problems has proven itself to be a challenging task.
- We have identified as a major culprit the tendency of novice Physics students to view the integral as an anti-derivative rather than a Riemann Sum.
- Past student experiences stress Calculus and formula memorization for calculating complicated integrals.

Our Solution

- To surpass this difficulty we adopted a pedagogy where simple summations substitute integration.
- We apply our pedagogy through online worksheets utilizing the Obojobo instructional platform.
- In our worksheets, students are first asked to compute approximate solutions to simple inhomogeneous problems by dividing the problem into subparts.
- Subsequently, the worksheets guide the students to obtain more accurate results by dividing the problem into a progressively larger number of subparts.
- Summation initially starts with one step containing a single term, and progressively evolves to sums with 3 or 4 terms: each time reducing the calculation error and approaching the exact result.
- No calculus is required, and integration ultimately arises as an infinite series of infinitesimally small terms.

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Introducing Integration in Physics without Calculus. Integrals for the layman.

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In this poster we describe a pedagogy on treating inhomogeneous problems in physics with integration without invoking calculus. The method consists of dividing the problem into a number of sub-problems, each treated as homogeneous and adding the partial results. The number of terms gradually increases each time, reducing the calculation error and approaching the exact result. Finally the integral appears as the summation limit to a Riemann Sum.

Integration in Physics Worksheet Sample

Problem

Computing the Voltage (Electric Potential) produced by a line charge distribution.

A rod of length L carries total charge Q uniformly distributed over its length. Therefore the rod charge density at the rod λ per unit length is given by the formula:

$$\lambda = \frac{dq}{dx} = \frac{Q}{L}$$

We want to calculate the Electric Potential (Voltage) generated by this charge distribution at point P located along the direction of the rod at distance L from the rod's right edge (see figure).

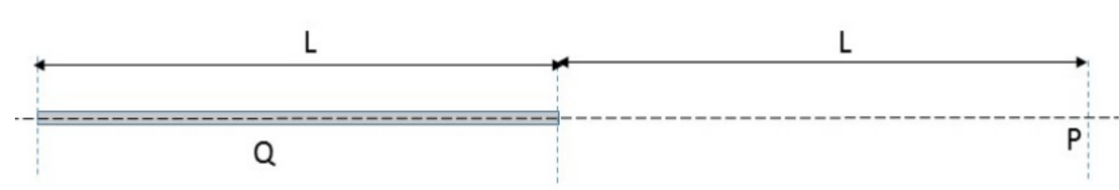

Fig 1

First divide the problem into 2 approximately homogeneous sub-problems (parts)

Split the rod into two equal parts (each having length L/2 and having charge Q/2). Assume that each of the two pieces is short enough that the voltage it produces can be accurately approximated by a point-like charge Q/2 positioned at the center of the segment.

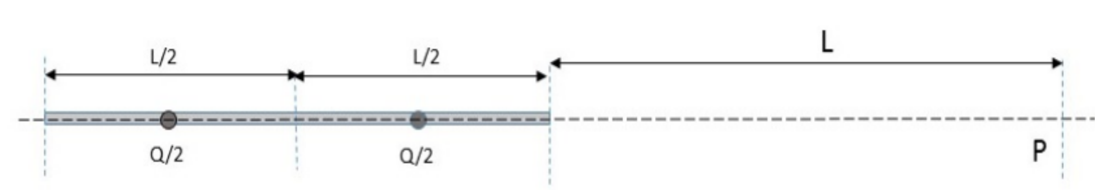

Fig 3

Practice

What is the distance of the first (leftmost) charge Q/2 from the point P?

Pick the correct answer

- 4/7
- 2/3

For better approximation, keep dividing the problem into a larger number of sub-parts, and treat each as homogeneous (answers are omitted for space)

Split the rod into three equal parts (each having length L/3 and having charge Q/3). Assume that each of the three pieces is short enough that the voltage it produces can be accurately approximated by a point-like charge Q/3 positioned at the center of its segment.

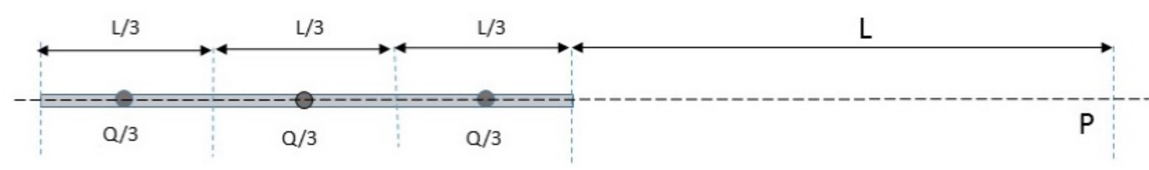

Fig 4

Practice

What is the distance of the first (leftmost) charge Q/3 from the point P?

Split the rod into four equal parts (each having length L/4 and having charge Q/4). Assume that each of the four pieces is short enough that the voltage it produces can be accurately approximated by the voltage produced by a point-like charge Q/4 positioned at the center of its segment.

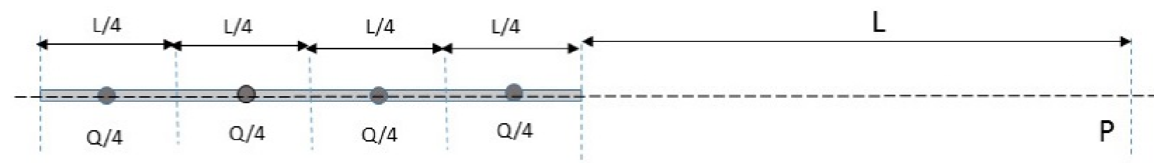

Fig 5

Practice

What is the distance of the first (leftmost) charge Q/4 from the point P?

We can repeat the above process as many times as we want. We can keep dividing the rod into smaller and smaller segments (with less and less length and charge). Theoretically, (for the mathematically inclined student) we can divide the rod into an infinite number of tiny segments. We symbolize the length of the infinitesimally small segment with dx.

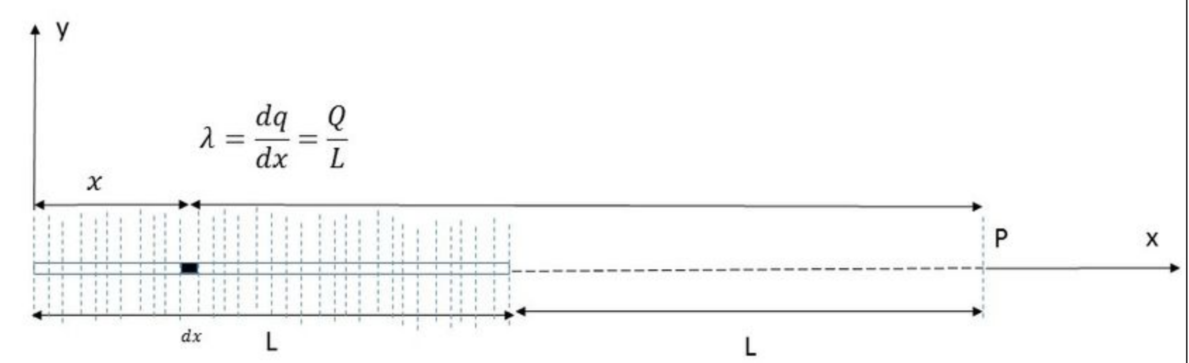

Fig 6

Do all required calculations and answer the following questions:

Practice

How large is the size (dx) of the infinitesimally small segment? (Hint: infinitesimally small)

Current Status and Future Plans

- We have developed a number of worksheets covering simple one-dimensional inhomogeneous problems for introductory Mechanics and Electromagnetism covered in Physics 1 and 2.
- In order to facilitate the dissemination of our pedagogy in large audiences (classes with more than 200 students) and in remote education classes, we have incorporated our worksheets into an online platform.
- Currently worksheets have been incorporated as multiple-choice questions without allowing free responses.
- Dissemination of the material as well as assessment of the pedagogy is planned for near future.

References

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